Evaluating the Predictive Ability of Seasonal Autoregressive Integrated Moving Average (SARIMA) Models When Applied to Food and Beverages Price Index in Kenya

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***Abstract* — Price instability has been a major concern in most economies. Kenya's commodity markets have been characterized by high price volatility affecting investment and consumer behaviour due to uncertainty on future prices. Therefore, precise forecasting models can help consumers plan for their expenditure and government policymakers formulate price control measures. Due to the seasonality of Kenya's food and beverage price indices, the current study postulates that the Seasonal Autoregressive Integrated Moving Average (SARIMA) model can best be the best fit model for the data. The study used secondary data on Kenya's monthly food and beverage prices index from January 1991 to February 2020 to examine the predictive ability of the possible SARIMA models based on the minimisation of the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). A first-order differenced SARIMA (1,1,1) (0,1,1)12 minimized these model evaluation criteria (AIC = 1818.15, BIC =1833.40). The cross-validation test results of 6, 12, 18, 24, 30, and 36 step-ahead forecasts demonstrated that SARIMA models are unstable for use in forecasting over a long-time period with a tendency of increasing prediction errors with an increase in the forecast period. It is anticipated that the findings of the current study will provide necessary valuable information to the policymakers and stakeholders to understand future trends in commodity price.**

***Keywords* — ARIMA, food and beverages, prediction, price index, SARIMA.**

I. INTRODUCTION

Price stability is the key objective of monetary policy formulation and implementation [1]. Reference [2] investigated the relationships between product prices and inflation following the sharp increases in food and oil prices between 2008 and 2011 using a Structural vector autoregressive (SVAR) and Granger causality techniques. Food and oil prices were major contributors to persistent inflationary pressure in Kenya. Due to the high price volatility in the commodity market, it is vital to develop price forecasting models to aid in making informed decisions about consumptions, investments, and government policy formulation [3]. As such, time series models have become useful tools in policymaking.

The Box and Jenkins' Seasonal Autoregressive Integrated Moving Average (SARIMA) is one of the popular models that have been used in time series modelling to capture the seasonality aspect of time series data [4] caused by aspects such as seasonality in production, as in the case of food prices. Reference [5] used an ARIMA (2, 1, 0)12 models to predict annual sugarcane yields in India from 1950 to 2012. In 2013, the fitted model predicted increased sugarcane yields but a significant decrease in 2014. Reference [6] identified that SARIMA (2,1,2) (2,0,3)4 minimized the Akaike Information Criteria (AIC) hence the best fit model to the quarterly sugarcane yields data for the period 1973 to 2014. They ascertained that SARIMA models suit a time series with seasonal patterns.

In Turkey, [7] recommended ARIMA (1,1,1) (1,0,2) [12] model with drift as the best model, among other competing SARIMA models, for forecasting monthly inflation rates (AIC–corrected (AICc) = 2405.96, Bayesian Information Criteria (BIC) = 2405.48). Based on the minimization of the Akaike Information Criterion (AIC) [8], [9] found that the SARIMA (2, 1, 1) x (1, 0, 1)12 is the best fit for the data from January 2003 to December 2016. The model minimised the prediction errors of tomato prices from January 2011 to December 2011 (MAPE = 125.251, RMSE = 32.063, and MAE = 22.3). The study found that the model is suitable for predicting future tomato prices in Nairobi County, Kenya. Given that commodity prices depict the seasonality component, this study evaluated the predictive ability of the

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SARIMA model using Kenya's monthly food and beverage prices index (FBPI) data.

II. Data and Methodology

A. Data

The study conveniently relied on Kenya's monthly FBPI data from January 1991 to February 2020 extracted from the Kenya National Bureau of Statistics (KNBS) website [10]. To test the predictive ability of the best-fit SARIMA model over a long-term horizon, the study adopted the cross-validation technique where the dataset is split into a training set (used to estimate the model) and a testing set (used to test the accuracy of the fitted model). The test period was 6, 12, 18, 24, 30, and 36 step ahead period forecasts. Thus, the study had six sets of test and training datasets. For the six steps ahead, the train set data spanned January 1991 to August 2019, whereas the test set spanned September 2019 to February 2020. Consequently, the starting dates for the test sets for the 12, 18, 24, 30, and 36 were January 2019, September 2018, January 2018, September 2017, and January 2017, respectively.

B. Time Series Modelling

Time series data frequently contain trend, seasonal, or cyclical components, leading to the emergence of statistical techniques used to model time series data [11]. Box and Jenkins pioneered the use of time series models (1960). Model selection, parameter estimation, and model diagnostics are iterative steps in the Box and Jenkins methodology.

1) Autoregressive Model

Reference [12] first introduced autoregressive (AR) models. The AR model dictates that a realization at time t is a sequence of the p preceding occurrences plus some noise term. Autoregressive models represent a given time series x_t in terms of its lagged observations. In a study by [12], an Autoregressive process of order p denoted as an AR(p) process, is defined as:

$$x_t = \mu + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + \beta_p x_{t-p} + e_t \tag{1}$$

Given that: μ denotes the mean value of the given series $x_t, \beta_i: i = 1,2,\ldots,p$ are the coefficients of the regression equation, e_t is a white noise process; $e_t \sim N(0,\delta^2)$ and if $p = 0, x_t = e_t$ with no autoregression term.

2) Moving Average Process

The Autoregressive (AR) models supplement Moving Average (MA) models [13]. The moving-average model (MA) is a popular approach for modelling univariate time series in time series analysis [14]. The MA model estimates the outcome variable as a linear combination of the current and various past values of a random term. In a study by [13], a moving average process of order q, represented as an MA(q) process, is defined as:

$$x_t = \mu + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_q e_{t-q} \tag{2}$$

Given that: μ denotes the mean of the given series x_t $\theta_i: i = 1,2,\ldots,q$ are the coefficients of the regression model and e_t is a white noise process; $e_t \sim N(0,\delta^2)$. Let the lag operator, denoted by B, is commonly used to express the process's lagged values usually defined by $B^k X_t = X_{t-k}$, then, we define:

$$\Phi(B) = 1 - \sum_{j=1}^{P} \emptyset_j B^j = 1 - \emptyset_1 B - \emptyset_2 B^2 - \emptyset_3 B^3 - \cdots - \emptyset_p B^p \tag{3}$$

The equation $\Phi(B)X_t = e_t, t = 1,2,\ldots,n$ gives the AR(p) process where; Φ(B) is referred to as the process's characteristic polynomial, and its roots ascertain whether or not the process is stationary.

3) Autoregressive Moving Average (ARIMA) Model

Reference [15] integrated both AR and MA processes into the ARMA processes could be used to model a wide range of stationary time series. This implies that a general time series x_t can be represented as a linear combination of previous x_t values and errors, e_t. The combination of the AR and MA models results in the ARMA (p, q) process, which is defined as;

$$x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} - \cdots - \phi_p x_{t-p} = e_t + \Theta_1 e_{t-1} + \Theta_2 e_{t-2} + \cdots + \Theta_q e_{t-q} \tag{4}$$

Which can be summarized as;

$$x_t = \sum_{i=1}^{p} \phi_i \, x_{t-i} + \sum_{j=1}^{q} \Theta_j \, e_{t-j} \quad (5)$$

Applying the backward shift operator, denoted by $B^k \; y_t = y_{t-k}$

$$[1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p] x_t = [1 - \Theta_1 B - \Theta_2 B^2 - \ldots - \Theta_q B^q] e_t \quad (6)$$

Therefore, we have $\phi(B)x_t = \Theta(B)e_t$. When $\phi(B) = 1$, then the model ARMA (p, q) becomes an MA (q) when $\Theta(B) = 1$, then the ARIMA (p, q) reduces to an AR process of order p that is AR (p). As a result, an ARMA process models a series in terms of its past values and errors. The data used in the AR, MA, and ARMA models must be stationary [16]. However, stationarity is corrected by differencing. Box and Jenkins, therefore, introduced Autoregressive Integrated Moving Average models (ARIMA), which utilize the differenced series.

4) Autoregressive Integrated Moving Average (ARIMA) Model

Reference [17] introduced ARIMA model, which describes a given process as a direct combination of its own lagged observations of order p and its lagged error terms of order q. Specifically; ARIMA can be decomposed into the following components:

1) AR part which uses past observations (x_{t-i}) to fit a series x_t with i^{th} an optimal lag order i.
2) I (Integrated). Making a time series stationary by using raw observation differencing.
3) MA part. A model applies the dependency between an observation and a residual error from a moving average model to lagged measurements.

Each of these elements is precisely defined as a parameter in the model. The ARIMA model with a specific order is known as the ARIMA (p, d, q) model, where the p, d, and q are integers greater or equal to zero, respectively [19]. The parameter, p, denotes the number of autoregressive lags, d denotes the order of integration that makes the data stationary, and q denotes the number of MA lags [20]. This model is suitable for non-stationary data, where a given series has to be differenced to be made stationary. In a study by Box and Jenkins on modelling, stationarity must be tested before fitting the model. A time series is stationary if its mean and variance are constant over time. Differentiating a non-stationary series into a stationary series entails computing the variation between consecutive observed values. The ARIMA class of models is a modification of ARMA class of models for modelling non-stationary time series data. The ARIMA (p, d, q) is described as [21]:

$$\Delta^d x_t = \beta_1 \Delta^d x_{t-1} + \beta_2 \Delta^d x_{t-2} + \ldots + \beta_p \Delta^d x_{t-p} + \sum_{j=1}^{q} \theta_j \, e_{t-j}, \quad (7)$$

Where: Δ^d is the differential operator of order d.
p denotes the order of the AR part.
q denotes the order of the MA part

The initial condition requires that: $\theta_0 = 1$. This model is suitable for a series that has been differenced to be made stationary and depicts linearity with its lagged values of order p and the error terms of order q [22]. ARIMA (p, d, q) represents a non-seasonal ARIMA model, where p denotes the number of AR lags, d gives the differencing lag, and q is the MA order, and it is given by;

$$X_t = \sum_{k=1}^{p} \alpha_k X_{t-k} + \sum_{j=1}^{q} \theta_j e_{t-j} + \mu_t + e_t, \quad (8)$$

This can be summarized by applying the backward shift operator as follows:

$$\alpha(B)X_t = \theta(B)e_t + \mu_t, \quad (9)$$

Where $\alpha(B) = 1 - \alpha_1 B - \alpha_2 B^2 - \ldots - \alpha_p B^p$ and $\theta(B) = 1 - \theta_1 B - \theta_2 \, B^2 - \ldots - \theta_q \, B^q$. If Z_t is a stationary series achieved by d differencing from the time series X_t, we obtain:

$$Z_t = \nabla^d X_t = (1 - B)^d X_t \quad (10)$$

As a result, the final form of the ARIMA (p, d, q) model could be shaped as follows:

$$(1 - B)^d \alpha(B) X_t = \theta(B) e_t \quad (11)$$

The above model accounts for past values that form the Autoregressive (AR) process and lagged values of the error terms forming the Moving-Average (MA) process. However, most time-series data, such as commodity prices like the food and beverage prices index, exhibit seasonal fluctuations over time. ARIMA models were therefore not suitable for modelling the time series data with seasonal fluctuations. For such a time series, SARIMA models are suitable. In a study by [23], the SARIMA model is suitable where a given time series data exhibit seasonality, thus appropriate for modelling food and beverage prices index with seasonal variations over time.

5) Seasonal Autoregressive Integrated Moving Average (SARIMA) Models

The SARIMA model extends the ARIMA model to fit seasonal time series data [24]. Seasonality is a regular pattern of repeated movements over time in a given time series [25]. The seasonality is depicted by most time-series data rendering the ARIMA model ineffective in forecasting a given series. The seasonal ARIMA (SARIMA) model combines the Autoregressive term and Moving Average term in an ARIMA model which predicts X_t using past values and noises with lags (h) as the periodicity of the series. The non-seasonal ARIMA part is denoted as ARIMA (p, d, q), where p is the order of the AR part, d is the order of differencing to make data stationary, and q is the order of the MA part. Thus, the SARIMA model combines non-seasonal and seasonal components in a multiplicative model and is abbreviated as ARIMA (p, d, q) (P, D, Q)h. where h denotes the number of seasons or the time span of the recurring seasonal pattern. The SARIMA model equation is expressed in (12) below.

$$\Phi(B^h)\varphi(B)X_t - \mu = \Theta(B^m)\Theta(B)e_t \tag{12}$$

Where $\Phi\ (B^h)$ is the seasonal AR process; $\varphi(B)X_t$ is the non-seasonal AR process; $\Theta(B^m)$ is the seasonal MA process; $\Theta(B)e_t$ is the non-seasonal MA process; X_t is the actual observation at time t; B is the backshift operator; e_t is the white noise and h is the periodicity of the seasonal component. The processes in (12) can be written using the backward shift operator as:

$$\begin{aligned} \Phi(B_h) &= 1 - \Phi_1 B^h - \Phi_2 B^{2h} - \cdots - \Phi_p B^{ph} \\ \varphi(B)Xt &= 1 - \varphi_1 X_{t-1} - \cdots - \varphi_P X_{t-p} \\ \Theta(B^m) &= 1 + \Theta B^m + \Theta_2 B_{2m} + \cdots + \Theta_q B_{qm} \\ \Theta(B)\ e_t &= 1 + \Theta_1 e_{t-1} + \cdots + \Theta_q e_{t-q} \end{aligned} \tag{13}$$

C. Stationarity and Invertibility

Stationarity and invertibility are the essential components when modeling with the ARIMA class of models and are discussed below.

1) Stationarity

In time series analysis, data must be evaluated for stationarity. If the mean and variance of a time series are constant, it is stationary. Testing for stationarity can be done using a time plot or the Augmented Dickey-Fuller (ADF) test [26]. Forecasting is challenging because time series are non-deterministic, implying that we cannot anticipate what will happen in the future with certainty. If the time series is found stationary, one can forecast that its statistical properties would be the same in the future as they were in the past. The stationarity condition guarantees that the estimated model's autoregressive parameters are steady within a given range and that the model's moving average parameters are invertible. The estimated model can be predicted if this condition is met [27]. To test for stationarity, we typically look for the presence or absence of a unit root. The series is considered stationary if the roots of the characteristic equation shown in (3) lie outside the unit circle. This is equivalent to saying that the estimated model's coefficients have absolute values less than one i.e., $|\emptyset_i| < 1$ for $i = 1,2, \ldots, p$. The current study adopts the Augmented Dickey-Fuller (ADF) test to test for the of unit root in the series. The stationarity is classified into two types: strictly stationary and weakly stationary.

a) Strict Stationarity

Reference [28] highlighted that a time series $\{X_t\ , t \in Z\}$ is thought to be strictly stationary when the joint distribution of $(X_{t_1}\ X_{t_2}\ X_{t_3} \ldots X_{t_k})$ is similar to that of $(X_{t_1}\ X_{t_2} X_{t_3} \ldots X_{t_k})$. In other words, strict stationarity implies that the joint distribution is determined solely by the "difference" h and not by the time $t_1, t_2, t_3, \ldots, t_k$.

b) Weak Stationarity

The time series $\{X_t\ , t\ \in\ Z\}$ is said to be weakly stationary if and only if the following conditions are met:

4) $E[{X_t}^2]\ <\ \infty, \forall t\ \in\ Z$
5) $E[X_t]\ = (constant)/\mu, \forall t\ \in\ Z$
6) $\gamma_X\ (s,t) = \gamma_X(s\ +\ h, t\ +\ h);\ \forall s,t,h\ \in\ Z.$

Additionally, a weakly stationary time series {X_t} should have three characteristics: finite variation, a constant first moment, and a second moment $\gamma_X(s,t)$ that relies only on $|t - s|$ rather than s or t [29]. Typically, the term stationary refers to a weakly stationary process, and when one wants to underscore that a process is strictly stationary, they will appropriately use strictly stationary.

2) Invertibility

The stationary condition applies only to the AR part. Such that if we have a seasonal and a non-seasonal AR (1) model, then stationary requires that $|\Phi_i| < 1$ and $|\varphi_i| < 1$. Unlike in the stationarity regulation, the invertibility condition applies to o the MA part of the SARIMA model. Thus, the parameter restrictions in (13) are such that $|\theta_i| < 1$ and $\Theta_i < 1$[7].

D. The Steps of Box Jenkins Methodology

The modelling process was carried out following the [17] methodology approach, which includes the stages of identification of the model, model specification, model estimation, and model diagnostics with preliminary steps of checking for stationarity of the data and the general specification of the class of the models. The steps are discussed as follows;

1) Model Identification

Given that commodity prices are likely to depict seasonality due to seasonality of production or supply chains, the current study postulates that the SARIMA model best fit the data under consideration. The SARIMA model is designed for a time series that captures seasonality. It combines non-seasonal and seasonal components in a multiplicative model and is abbreviated as ARIMA (p, d, q) (P, D, Q)h hence it is essential to select or specify the optimal model as discussed below.

2) Model Selection and Specification Criteria

The model selection entails identing the best fit model to a given data. The principle of parsimony which holds that the model with least parameters is favoured and used [30]. The first and initial step of modelling is identifying the suitable order of the SARIMA (p, d, q) (P, D, Q)h model. The order specification and selection of orders p and q entail plotting ACF and PACF or a correlogram of variables at various lag lengths, while the order d is determined based on the Integration (1) or Integration (0) process [31].

These orders are determined by the stationary series' sample autocorrelation function (ACF) and partial autocorrelation function (PACF). The ACF expresses the internal correlation between observations in a time series separated by different lengths as a function of time lag [32]. These two plots indicate the type of model we should create.

When inspecting the ACF and PACF plots, it is important to consider seasonal and non-seasonal lags. Tables I and II show how the ACF and PACF behave for seasonal and non-seasonal series [33]. The values of d and D are determined by the order of differencing performed to make the series stationary at the no-seasonal and seasonal levels, respectively.

TABLE I: ACF AND PACF BEHAVIOUR FOR NON-SEASONAL ARIMA (P, Q)

	AR(p)	MA(q)	ARMA(p,q)
ACF	Tail off at lag k	Cuts off at lag q	Tails off
PACF	Cuts off at lag p	Tails off at lag k	Tails off

TABLE II: ACF AND PACF BEHAVIOUR FOR THE SEASONAL ARIMA (P, Q)

	AR(P)	MA(Q)	ARMA(P,Q)
ACF	Tail off at lag K	Cuts off at lag Q	Tails off
PACF	Cuts off at lag P	Tails off at lag K	Tails off

3) Model Estimation

After identifying the model, the next step in SARIMA model construction is the estimation of chosen model parameters. The maximum likelihood estimation (MLE) method was used to estimate the parameters. Since the past lagged observations of the noise terms cannot be observed in a SARIMA model, the MLE method is preferred over Ordinary Least Squares (OLS) regression analysis. Given T observed terms, the log-likelihood (LL) function $\{y_t(\theta)\}$ is signified by:

$$Log\ L\,[y_t(\theta)] = \sum_{t=1}^{T}\left[ln[D(z_t(\theta), v)] - 0.5\,[ln\,(\sigma^2{}_t(\theta))]\right] \quad (14)$$

Where; θ is a vector of the conditional mean, conditional variance, and density function parameters, and is an independent and identically distributed (iid) random variable with zero mean and variance of one.The Multivariate Normal Distribution (MND) with a given mean and covariance matrix is used to estimate the parameters of θ denoted by:

$$V_n = \frac{1}{n}(Var[V_\theta ln(f_x(X;\theta_0))])^{-1} \quad (15)$$

Where: $ln(f_x\ (X;\theta_0)$ gives the log-likelihood of one observation from the sample, estimated at the parameter θ_0. $V_\theta\ ln(f_x\ (X;\theta_0)$ is the vector of the first derivatives of the log-likelihood. The Outer Product of Gradients (OPG) estimate is used to estimate the asymptotic covariance matrix, and it is calculated as:

$$\widehat{V_n} = \left\{\frac{1}{n}\sum_{i=1}^{n} V_\theta ln\left(f_x(X;\hat{\theta}_n)\right)\right\} V_\theta ln(f_x(X;\theta_0))^{-1} \quad (16)$$

4) Model Diagnostics

Following the estimates of the ARIMA model's parameters, the next step in the Box-Jenkins model building approach is to check the model's adequacy, also known as model diagnostics. A model should extract all systematic properties from the data. The residuals (the portion of the data that the model does not explain) should be small [34]. Thus, the diagnostic check is built on the model's residuals. One premise is that an adequate model's residuals ought to be white noise. If e_t is a sequence of iid random variables with discrete mean and variance; the noise is said to be white. If e_t is normally distributed with a mean of zero and a variance of σ_t, the series is referred to as Gaussian White Noise [35]. Another diagnostic check is that of autocorrelation. A correlogram can be used to test the autocorrelation of the residuals. If there is no serial correlation, autocorrelations and partial autocorrelations at all lags should be close to zero. A statistical tool such as the Ljung Box Q statistic can be used to establish serial independence. In this case, the Box-Pierce Q statistics and Ljung-Box LB statistics are used to determine the significance level of individual coefficients. In the study by [36], the Box-Pierce Q statistics is denoted as;

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{{e_k}^2}{n-2} \quad (17)$$

Where e_k implies the residual autocorrelation at given lag k, n denotes the number of residuals, and m represents time lags. If there is no serial and all Q-Statistics ought to be insignificant [37]. Normality and homoscedasticity tests among the residuals are evaluated using statistical tools like the Autoregressive Conditional Heteroskedastic – Lagrange multiplier (ARCH–LM) test can be used [38].

5) Forecasting Using SARIMA

Forecasting is ultimate goal of any time series model. It is essential in the decision-making process. It is also a planning tool, facilitating decision-makers to anticipate future uncertainty based on previous and current observations' behaviour. Reference [18] defined forecasting as the foundation for business and economic planning, inventory and production control, and industrial process enhancement. Forecasting is the process of making predictions about unknown quantities. It is the practice of developing statements about events whose results have yet to be seen. After passing the diagnostic checks for its adequacy, the model can then be used for forecasting. For example, given the Seasonal ARIMA $(0,1,1)(1,0,1)_{12}$, the next step ahead forecasts are as follows: [11].

$$Z_t = Z_{t-1} + \Phi(Z_{t-12} - Z_{-13}) + \varepsilon_t - \theta\varepsilon_{t-1} - \Phi\varepsilon_{t-12} + \theta\varepsilon_{t-13} \quad (18)$$

The forecast one step ahead from the origin t is provided by:

$$\hat{Z}_{t+1} = Z_t + \Phi(Z_{t-11} - Z_{-12}) - \theta\varepsilon_t - \Phi\varepsilon_{t-11} + \theta\varepsilon_{t-12} \quad (19)$$

The next two step is

$$\hat{Z}_{t+2} = Z_{t+1} + \Phi(Z_{t-10} - Z_{-11}) - \Phi\varepsilon_{t-10} + \theta\varepsilon_{t-11} \quad (20)$$

and so on. The residual terms $\varepsilon_{13}, \varepsilon_{12}, \varepsilon_{11}, \ldots, \varepsilon_1$ enters the predictions for leading times denoted by $l = 1,2,\ldots,13$ however, for l greater than 13, the AR part takes over.

$$\hat{Z}_{t+1} = Z_{t-l+1} + \Phi Z_{t+l-12} - \Phi Z_{t+l-13}\ for\ l > 13 \quad (21)$$

III. Results

A. Testing for Non-stationarity

The data analysis results presented in this section was done using R [39]. Fig. 1 shows the time plot of the food and beverage index data from January 1991 to February 2020 (observed) alongside the associated three components and random error (seasonal, trend, and random error). There is an overall increasing trend in the series with two peaks linked with the 2008/09 global financial crisis and the 2007/08 post-election violence. Given that the series has a seasonal component, the SARIMA model is appropriate since it captures seasonality.

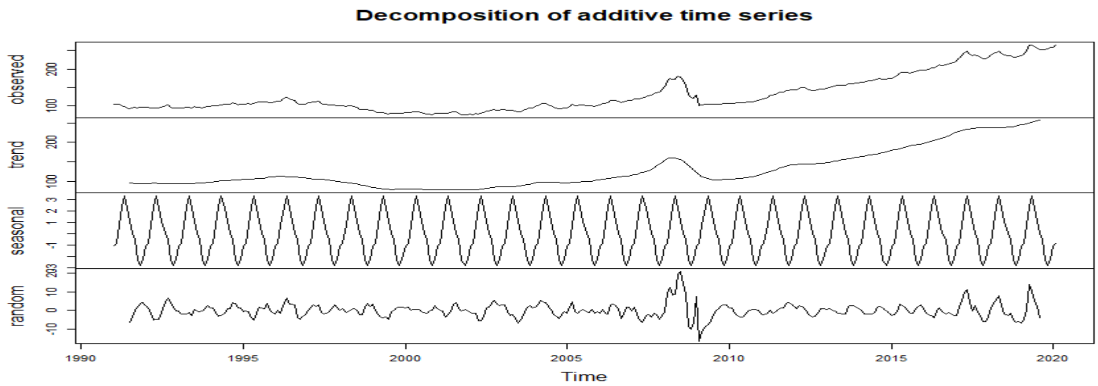

Fig. 1. Decomposition of Kenya's FBPI series.

Fig. 1 depicts the monthly series of FBPI data evolution. The ADF unit roots test showed non-stationarity of the series at level (ADF = - 0.87, p = 0.95). The first differenced (ADF = -7.89, p = 0.01), seasonal differenced series of lag 12 (ADF = - 6.31, p = - 0.01) were stationary. The finding implies that D = 0 in the Seasonal ARIMA (P, D, Q) model, whereas d = 1 in the non-seasonal ARIMA case. However, this theoretical aspect is not quantifiable in forecasting. Thus, the selection criteria adopted are discussed in section 3.2

B. Model identification

While the ACF and PACF can be used to select the order of the best fit model, it is not always accurate. Thus, a grid search of possible orders was employed to identify the most parsimonious model out of a combination of orders with the seasonal and non-seasonal parts. The selection of the best order of these components was based on the minimization of AIC and BIC. The best order among the 17 competing SARIMA orders was SARIMA (1,1,1) (0,1,1)12 (AIC = 1818.150, BIC =1833.40) (Table III).

TABLE III: Evaluation of SARIMA Models Using AIC and BIC

SARIMA MODEL	AIC	BIC
SARIMA (0,1,2) $(1,0,0)_{12}$	1860.97	1876.39
SARIMA (0,1,2) $(0,0,1)_{12}$	1860.97	1876.39
SARIMA (0,1,2) $(0,1,1)_{12}$	1820.37	1835.63
SARIMA (0,1,3) $(1,0,1)_{12}$	1858.77	1881.90
SARIMA (1,1,0) $(1,0,0)_{12}$	1858.57	1870.13
SARIMA (1,0,0) $(0,1,1)_{12}$	1818.17	1829.64
SARIMA (1,1,0) $(0,1,1)_{12}$	1818.17	1829.64
SARIMA (1,1,1) $(0,0,1)_{12}$	1859.08	1874.50
SARIMA (1,1,1) $(0,1,1)_{12}$	1818.15	1833.43
SARIMA (1,1,2) $(1,0,0)_{12}$	1860.91	1880.18
SARIMA (2,1,0) $(1,0,0)_{12}$	1859.30	1874.72
SARIMA (2,1,1) $(1,0,0)_{12}$	1859.70	1879.00
SARIMA (2,1,1) $(0,0,1)_{12}$	1859.72	1879.00
SARIMA (3,1,0) $(1,0,0)_{12}$	1861.19	1880.46
SARIMA (3,1,1) $(1,0,0)_{12}$	1861.71	1884.84
SARIMA (3,1,1) $(0,0,1)_{12}$	1862.87	1886.00
SARIMA (0,1,3) $(1,0,0)_{12}$	1862.97	1882.24

C. Model Estimation

The MLE method was used to estimate the model coefficients. The SARIMA (1, 1, 1) $(0, 1, 1)_{12}$ includes one non-seasonal and seasonal AR processes, one non-seasonal MA process, one non-seasonal difference (d = 1), and one seasonal difference (D =1). The resultant parameter estimates for the fitted model were; $\varphi_1 = 0.5755$, $\Theta_1 = -0.3045$, $\Theta_1 = -0.921$. The resultant SARIMA (1,1,1) $(0,1,1)_{12}$ model is as expressed in (22) below

$$X_t = 0.576\, X_{t-1} - 0.305\, e_{t-1} - 0.921\, \epsilon_{t-1} \tag{22}$$

Where; X_t is the food and beverage price index at time t, X_{t-1} is the non-seasonal AR (1) process, e_{t-1} is the non-seasonal Moving average process of order 1, and ϵ_{t-1} denotes the seasonal MA (1) process.

D. Model Diagnostics Check.

Two diagnostic tests were performed, particularly evaluating if the fitted model's residuals meet the assumptions of normality and autocorrelation. The two robustness checks are discussed below.

1) Normality Test

The model residuals should be iid sequence with zero mean and constant variance. The Q-Q plot in Fig. 2 show that the residuals are approximately normally distributed since they lie along the 45^0 line.

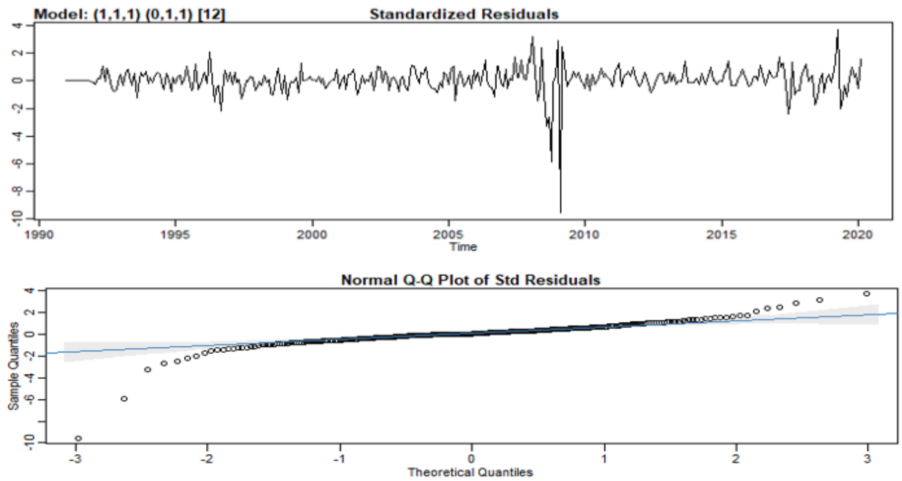

Fig. 2. The fitted model's residual plot and the resultant Q-Q plot.

2) Autocorrelation

The ACF plot of the residuals in Fig. 3 shows insufficient evidence of significant spikes. Thus, there is no significant autocorrelation and partial autocorrelation coefficients between residuals and given past residuals. The finding is supported by the Ljung–Box Q-test which shows insignificant Q-statistics (all $p > 0.05$). The finding suggests that residuals are purely white noise hence there is no need to fit the ARCH model and that the fitted SARIMA (1,1,1) (0,1,1)12 model can therefore be used for forecasting. The findings imply that residuals errors are not correlated and thus do not require ARCH class of models.

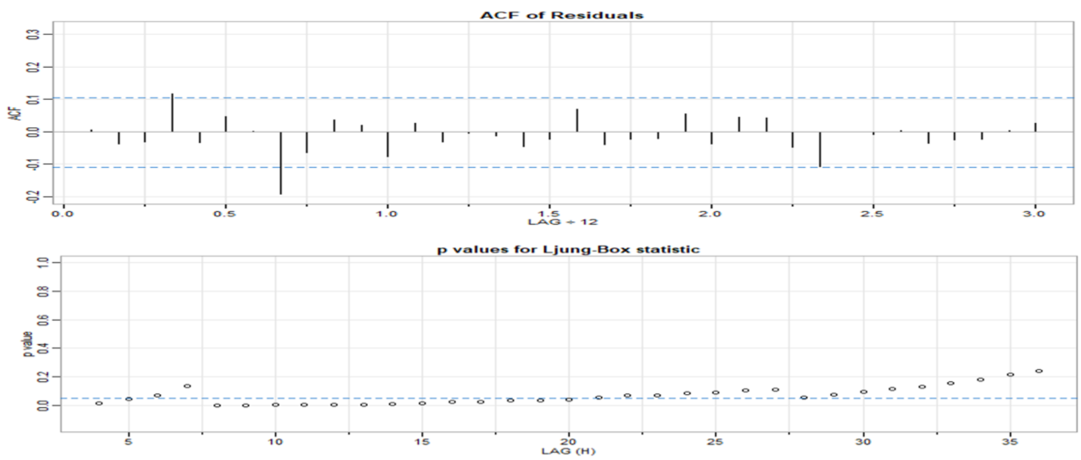

Fig. 3. The Ljung–Box Q-test and the ACF of model residuals.

E. Predictive Ability of SARIMA Models

As much as SARIMA models can be good candidates for modelling time series data, their consistency in fitting data over a long-time horizon can be questioned. The test can be done using cross-validation, where the dataset is split into a training set (used to estimate the model) and a testing set (used to test the accuracy of the fitted model). The approach suitably allows one to compare accuracy over a given time horizon. Since predictive analytics looks into the future, the first n-sample points formed the hold-out sample proportional to the h-step ahead periods. The accuracy metrics for the next 36 months are presented in Table IV. The results show that SARIMA models are unstable for forecasting over a long-time period. As indicated by RMSE, MAE, and MAPE, the prediction errors are relatively small in the 6-step ahead compared to 12, 18, 24, 30, and 36 step ahead period forecasts.

TABLE IV: ACCURACY METRICS OF H-STEP AHEAD FORECASTS

	h-step ahead forecasts					
Parameter	6	12	18	24	30	36
Start period	Sept 2019	Jan 2019	Sept-2018	Jan-2018	Sept 2017	Jan-2017
ARIMA (S=12)	(1,1,0) (1,1,2)	(1,1,1) (1,1,0)	(1,1,1) (1,1,0)	(1,1,0) (1,1,0)	(1,1,0) (1,1,0)	(1,1,1) (1,1,0)
RMSE	2.541	13.243	10.007	9.987	20.072	20.389
MAE	1.944	12.225	7.779	8.813	18.147	16.916
MAPE	0.769	4.730	3.021	3.603	7.497	7.164

Note: Model orders are autogenerated from the auto. Arima () function in R based on the minimization of AIC and BIC; The accuracy metrics show the deviation of the forecasted values from the actual observation in the h-step ahead period.

Generally, there is a tendency of increasing prediction errors with an increase in the forecast period. Nonetheless, the model is pertinent to the monetary policy committee (MPC) carrying out the frequent intermittent evaluation of commodity prices, say quarterly, to guide their policy options like price control.

F. Forecasting

The practice of forecasting helps in the planning and decision-making process since it gives an insight into the future uncertainty [40]. Given that the model is resilient over a short-term period, the predictions for the next 6 months from March 2021 to February 2021 alongside the one and two standard deviations prediction bounds from the mean are overlaid in the dark and a light grey area, respectively (Fig. 4). The forecast results depicted an increasing trend of FBPI from 269.89 in March 2012 to 273.95 in May 2021, followed by a decreasing trend to 269.52 in November 2020. Thereafter, the models predict a rising to 273.92 as of February 2021. The FBPI shows an overall increasing trend over time and is likely to keep rising in the future. The study finding is consistent with the past studies of [34], [41] and [42].

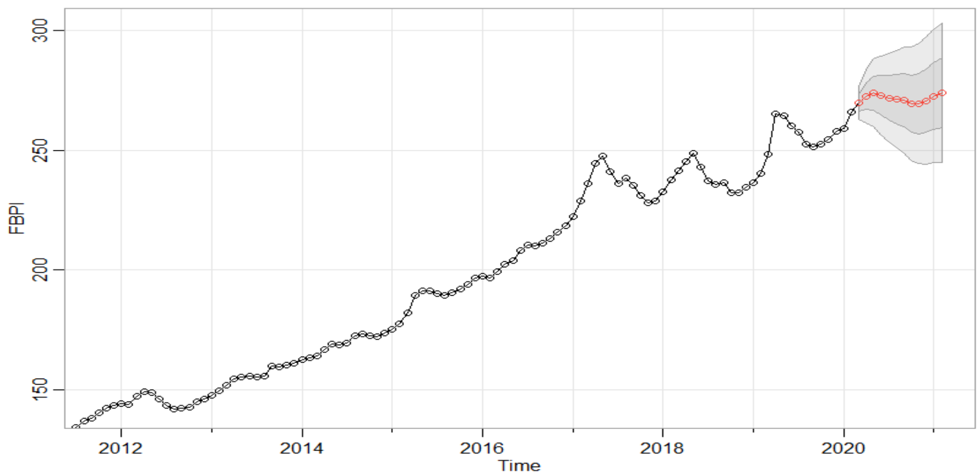

Fig. 4. Actual time series plot of FBPI and the 24-step ahead period forecasts.
Note: The predicted values are in red with the 95 percent and 99 percent confidence intervals in dark and light grey areas, respectively

IV. CONCLUSION

As a forward-looking practice in policy formulation, the Central banks design and implement monetary policy actions such as interest rate setting and price controls considering the future course of market prices. As such, forecasting models have become essential in policy formulation. Following the Box-Jenkins approach, this study evaluated the predictive ability of the SARIMA models using the Kenya's monthly FBPI data from January 1991 to February 2020. The findings showed that the best order among the 17 competing SARIMA models was SARIMA (1,1,1) (0,1,1)12 (AIC = 1818.150, BIC =1833.40). The cross-validation technique of evaluating the resilience of a model over a short- and long-term horizon demonstrated that SARIMA models can be unstable for use over a long period due to increasing prediction errors with an increase in the forecast period. Thus, SARIMA models are appropriate in making short-run forecasts. The model is pertinent to the MPC who carry out the quarterly assessment of price indices to direct their policy actions, like the price controls.

Recommendation From This Study

The current study also carries important implications for practice. Since the FBPI depicts an upward trend, the MPC should control inflation using fiscal tools such as capping interest rates, reduced government expenditure, and sustainable taxes. There is also a dire need to strengthen food security, such as creating a buffer stock of grains during great harvest to mitigate volatility in the global market supply. Trade liberalization in regional and global agricultural products markets can also prevent tariffs-related costs that push food prices. Lastly, international environmental protection has been global warming associated with increased desertification. The government of Kenya ought to adopt advanced and sustainable agricultural development and innovation in semi-arid and arid areas to supplement the food supply in the country.

Further Research

Future studies can consider a hybrid model such as SARIMA and Support Vector Regression (SVR) that include covariates to enhance the model's prediction of the consumer price index. The SVR model can incorporate consumer price index predictors such as interest rates and taxes to account for more variation in the commodity prices.

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Conflict of Interest

The authors declare no competing interests.

References

[1] CBK Central Bank of Kenya – Monetary Policy Statement, CBK, Nairobi. [Internet] 2017 [cited 2021 January 17] Available from: https://www.centralbank.go.ke/uploads/monetary_policy_statements/2039096141_41st%20Monetary%20Policy%20Statement,%20December%202017.pdf

[2] Misati RN, Nyamongo EM, Mwangi I. Commodity price shocks and inflation in a net oil-importing economy. *OPEC Energy Review.* 2013; 37(2): 125-48.

[3] Knaut A, Paschmann M. Price volatility in commodity markets with restricted participation. *Energy Economics.* 2019; 81: 37-51.

[4] Saz G. The efficacy of SARIMA models for forecasting inflation rates in developing countries: The case for Turkey. *International Research Journal of Finance and Economics.* 2011; 62: 111-42.

[5] Kumar M, Anand M. An application of time series ARIMA forecasting model for predicting sugarcane production in India. *Studies in Business and Economics.* 2014; 9(1): 81-94.

[6] Mwanga D, Ong'ala J, Orwa G. Modeling sugarcane yields in the Kenya sugar industry: A SARIMA model forecasting approach. *International Journal of Statistics and Applications.* 2017; 7(6): 280-8.

[7] Özmen M, ŞANLI S. Detecting the best seasonal arima forecasting model for monthly inflation rates in turkey. *Dokuz Eylül Üniversitesi İktisadi İdari Bilimler Fakültesi Dergisi.* 2017; 32(2): 143-82.

[8] Akaike H. On entropy maximization principle, Applications of Statistics. InProceedings of the Symposium held at Wright State University, PR Krishnaiah, Ed., North-Holland Publishing Company 1977: 27-41.

[9] Mutwiri RM. Forecasting of tomatoes wholesale prices of Nairobi in Kenya: time series analysis using SARIMA model. *International Journal of Statistical Distributions and Applications*. 2019; 5(3): 46.

[10] ECONOMIC SURVEY. Kenya National Bureau of Statistics. [Internet] 1994. [cited 2021 March 20] Available from: www.knbs.or.ke/?wpdmpro=economic-survey-1994-2. Available from: https://www.knbs.or.ke/?wpdmpro=economic-survey-1994-2

[11] Cryer JD, Chan KS. Time series analysis with applications in R second edition Springer science+ business media. [Internet] 2008. [cited 2021 October 14] Available from: https://link.springer.com/book/10.1007/978-0-387-75959-3

[12] Makridakis S, Hibon M. ARMA models and the Box–Jenkins methodology. *Journal of Forecasting.* 1997; 16(3): 147-63.

[13] Qin D, Gilbert CL. The error term in the history of time series econometrics. *Econometric Theory.* 2001; 17(2): 424-50.

[14] Guin A. Travel time prediction using a seasonal autoregressive integrated moving average time series model. In2006 IEEE Intelligent Transportation Systems Conference 2006: 493-498.

[15] Zhang GP. Time series forecasting using a hybrid ARIMA and neural network model. *Neurocomputing.* 2003; 50: 159-75.

[16] Ding F, Shi Y, Chen T. Performance analysis of estimation algorithms of nonstationary ARMA processes. *IEEE Transactions on Signal Processing.* 2006; 54(3): 1041-53.
[17] Box GE, Jenkins GM, Reinsel GC. Time Series Analysis, Forecasting and Control, 3rd Edn (Englewood Cliþs, NJ, Prentice-Hall 1994. [Internet] 1994. [cited 2021 December 20] Available from: https://www.academia.edu/download/30152084/176871241.pdf
[18] Ziegel E, Box G, Jenkins G, Reinsel G. Time Series Analysis, Forecasting, and Control. *Technometrics*. 1995; 37(2): 238.
[19] Khashei M, Bijari M. An artificial neural network (p, d, q) model for timeseries forecasting. *Expert Systems with Applications.* 2010; 37(1): 479-89.
[20] Kleiber C, Zeileis A. Time Series. InApplied Econometrics with R. Springer, New York. 2008: 1-31.
[21] Muriithi DK, Ngeretha AN, Muriungi RG, Njoroge EW. Analysis of the Fluctuation of Gross Domestic Product In Kenya Using Autoregressive Integrated Moving Average Model. *Journal of Statistics: Advances in Theory and Applications.* 2014; 11(1): 31-43.
[22] Permanasari AE, Hidayah I, Bustoni IA. SARIMA (Seasonal ARIMA) implementation on time series to forecast the number of Malaria incidence. In2013 International Conference on Information Technology and Electrical Engineering (ICITEE). 2013: 203-207.
[23] Martinez EZ, Silva EA, Fabbro AL. A SARIMA forecasting model to predict the number of cases of dengue in Campinas, State of São Paulo, Brazil. *Revista da Sociedade Brasileira de Medicina Tropical*. 2011; 44(4): 436-40.
[24] Shekhar S, Williams BM. Adaptive seasonal time series models for forecasting short-term traffic flow. *Transportation Research Record.* 2007; 2024(1): 116-25.
[25] Verbesselt J, Hyndman R, Newnham G, Culvenor D. Detecting trend and seasonal changes in satellite image time series. *Remote Sensing of Environment*. 2010; 114(1): 106-15.
[26] Lee CC, Chang CP. New evidence on the convergence of per capita carbon dioxide emissions from panel seemingly unrelated regressions augmented Dickey–Fuller tests. *Energy.* 2008; 33(9): 1468-75.
[27] Hamilton JD. State-space models. Handbook of econometrics. 1994; 4: 3039-80.
[28] Francq C, Zakoïan JM. Strict stationarity testing and estimation of explosive and stationary generalized autoregressive conditional heteroscedasticity models. *Econometrica.* 2012; 80(2): 821-61.
[29] Lindner AM. Stationarity, mixing, distributional properties and moments of GARCH (p, q)–processes. InHandbook of financial time series. Springer, Berlin, Heidelberg. 2009: 43-69.
[30] Vandekerckhove J, Matzke D, Wagenmakers EJ. 14 Model Comparison and the Principle of Parsimony. The Oxford Handbook of Computational. 2015: 300.
[31] Nkoro E, Uko AK. Autoregressive Distributed Lag (ARDL) cointegration technique: application and interpretation. *Journal of Statistical and Econometric Methods.* 2016; 5(4): 63-91.
[32] Devi BU, Sundar D, Alli P. An effective time series analysis for stock trend prediction using ARIMA model for nifty midcap-50. *International Journal of Data Mining & Knowledge Management Process.* 2013; 3(1): 65.
[33] Shumway RH, Stoffer DS, Stoffer DS. Time series analysis and its applications. [Internet] 2000. [cited 2021 July 23] Available from: http://www.stat.ucla.edu/~frederic/221/W21/tsa4.pdf
[34] Gikungu SW, Waititu AG, Kihoro JM. Forecasting inflation rate in Kenya using SARIMA model. *American Journal of Theoretical and Applied Statistics.* 2015; 4(1): 15-8.
[35] McLeod AI. Parsimony, model adequacy and periodic correlation in time series forecasting. *International Statistical Review/Revue Internationale de Statistique.* 1993: 387-93.
[36] Horowitz JL, Lobato IN, Nankervis JC, Savin NE. Bootstrapping the Box–Pierce Q test: a robust test of uncorrelatedness. *Journal of Econometrics.* 2006; 133(2): 841-62.
[37] Dritsaki C. Forecast of SARIMA models: An application to unemployment rates of Greece. *American Journal of Applied Mathematics and Statistics.* 2016; 4(5): 136-48.
[38] Racine JS, Maasoumi E. A versatile and robust metric entropy test of time-reversibility, and other hypotheses. *Journal of Econometrics.* 2007; 138(2): 547-67.
[39] R Core Team. R: A Language and Environment for Statistical Computing. Vienna, Austria. [Internet] 2016. [cited 2021 May 23] Available from: https://www.R-project.org/
[40] Doulah MS. Forecasting Temperatures in Bangladesh: An Application of SARIMA Models. *International Journal of Statistics and Mathematics.* 2018; 5(1): 108-18.
[41] Nyoni T. Modeling and forecasting inflation in Kenya: Recent insights from ARIMA and GARCH analysis. *Dimorian Review.* 2018; 5(6): 16-40.
[42] Uwilingiyimana C, Munga'tu J, Harerimana JD. Forecasting Inflation in Kenya Using Arima-Garch Models. *International Journal of Management and Commerce Innovations.* 2015; 3(2): 15-27.